Project Euler #216: Investigating the primality of numbers of the form 2n² - 1

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This problem is a programming version of Problem 216 from projecteuler.net

Consider three integers a, b and c where $a > 0$, $\gcd(a, b, c) = 1$ and $b^2 - 4ac$ is not the square of an integer.

Let the second degree polynomial $P = aX^2 + bX + c$. In this challenge, we will be interested in the prime values of $P(n)$ for integers $n \geq 0$.

E.g. with $a = 2$, $b = 0$ and $c = -1$, the first such prime numbers are 7, 17, 31, 71, 97, 127 and 199.

How many numbers $P(n)$ are prime for $0 \leq n \leq N$?

Input Format

The first line of each test case contains three space-separated integers a, b and c.
The second line contains a single integer q which is the number of queries.
Each of the next q lines contains a value of N.

Constraints

- $1 \leq q \leq 10^5$.
- $a \in \{1, 2\}$.
- $|b| \leq 100$.
- $|c| \leq 10^7$.
- $\gcd(a, b, c) = 1$ and $b^2 - 4ac$ is not a perfect square.
- $0 \leq N \leq 10^7$.

Output Format

Print the answer to each query in a new line.

Sample Input 0

```
2 0 -1
1
10
```

Sample Output 0

```
7
```

Explanation 0

The values of $P(n) = 2n^2 - 1$ for $0 \leq n \leq 10$ are :

$$[-1, 1, 7, 17, 31, 49, 71, 97, 127, 161, 199]$$

Only $[7, 17, 31, 71, 97, 127, 199]$ are prime. Hence the answer is 7.

Sample Input 1

```
2 0 1
1
20
```

Sample Output 1

```
4
```

Explanation 1

The evaluation of $P(n) = 2n^2 + 1$ for $0 \leq n \leq 20$ yields to :

$$[1, 3, 9, 19, 33, 51, 73, 99, 129, 163, 201, 243, 289, 339, 393, 451, 513, 579, 649, 723, 801]$$

The prime values in this list are $[3, 19, 73, 163]$. Therefore the answer is 4.

Sample Input 2

```
1 0 1
1
13
```

Sample Output 2

```
5
```

Explanation 2

There exist 5 prime numbers of the form $n^2 + 1$ where $0 \leq n \leq 13$: $[2, 5, 17, 37, 101]$.