Project Euler #212: Combined Volume of Cuboids

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This problem is a programming version of Problem 212 from projecteuler.net

An *axis-aligned cuboid*, specified by parameters $\{ (x_0, y_0, z_0), (dx, dy, dz) \}$, consists of all points (X, Y, Z) such that $x_0 \le X \le x_0 + dx$, $y_0 \le Y \le y_0 + dy$, $z_0 \le Z \le z_0 + dz$. The volume of the cuboid is the product, $dx \times dy \times dz$. The *combined volume* of a collection of cuboids is the volume of their union and will be less than the sum of the individual volumes if any cuboids overlap.

Let $C_1, \dots, C_N$ be a collection of N axis-aligned cuboids such that C_n has parameters

$x_0 = S_{6n-5}$ modulo M_x
$y_0 = S_{6n-4}$ modulo M_y
$z_0 = S_{6n-3}$ modulo M_z
$dx = 1 + (S_{6n-2}$ modulo $D_x)$
$dy = 1 + (S_{6n-1}$ modulo $D_y)$
$dz = 1 + (S_{6n}$ modulo $D_z)$

where $S_1, \dots, S_{6N}$ come from the "Lagged Fibonacci Generator":

For $1 \le k \le 55$, $S_k = (100003 - 200003k + 300007k^3)$ modulo 10^6
For $56 \le k$, $S_k = (S_{k-24} + S_{k-55})$ modulo 10^6

For example, if $M_x = M_y = M_z = 10^4$ and $D_x = D_y = D_z = 399$, then C_1 has parameters $\{(7, 53, 183), (94, 369, 56)\}$, C_2 has parameters $\{(2383, 3563, 5079), (42, 212, 344)\}$, and so on.

With such M_x, M_y, M_z, D_x, D_y, and D_z, the combined volume of the first 100 cuboids, $C_1, \dots, C_{100}$, is 723581599.

What is the combined volume of N cuboids, $C_1, \dots, C_N$?

Input Format

The only line of each test file contains exactly seven space-separated integers: $M_x, M_y, M_z, D_x, D_y, D_z$, and N.

Constraints

$2 \le M_x, M_y, M_z, D_x, D_y, D_z, N \le 10^5$

Output Format

Print exactly one number: the combined volume of N cuboids.

Sample Input 0

```
53 54 48 257 51 81 2
```

Sample Output 0

```
88970
```

Explanation 0

With the given $M_x = 53,\ M_y = 54,\ M_z = 48,\ D_x = 257,\ D_y = 51,$ and $D_z = 81,$ the cuboid C_1 has parameters { $(38, 45, 39), (61, 33, 38)$ } and the cuboid C_2 has parameters { $(38, 45, 39), (62, 35, 41)$ }.

It is clear that the cuboid C_1 is within the boundaries of the cuboid C_2 therefore the combined volume of the two cuboids equals to the volume of the second cuboid which is

$62 \times 35 \times 41 = 88970.$

Sample Input 1

```
4649 7681 6382 113 75 93 2
```

Sample Output 1

```
538384
```

Explanation 1

With the given $M_x = 4649,\ M_y = 7681,\ M_z = 6382,\ D_x = 113,\ D_y = 75,$ and $D_z = 93,$ the cuboid C_1 has parameters { $(100, 200, 275), (76, 39, 92)$ } and the cuboid C_2 has parameters { $(2662, 3710, 5079), (76, 38, 92)$ }.

With such small cuboid sizes, it is clear that the cuboids have no overlap therefore the combined volume of the two cuboids equals to the sum of their volumes which is

$76 \times 39 \times 92\ +\ 76 \times 38 \times 92 = 538384.$

Sample Input 2

```
10000 10000 10000 399 399 399 100
```

Sample Output 2

```
723581599
```

Explanation 2

As noted in the problem statement.