

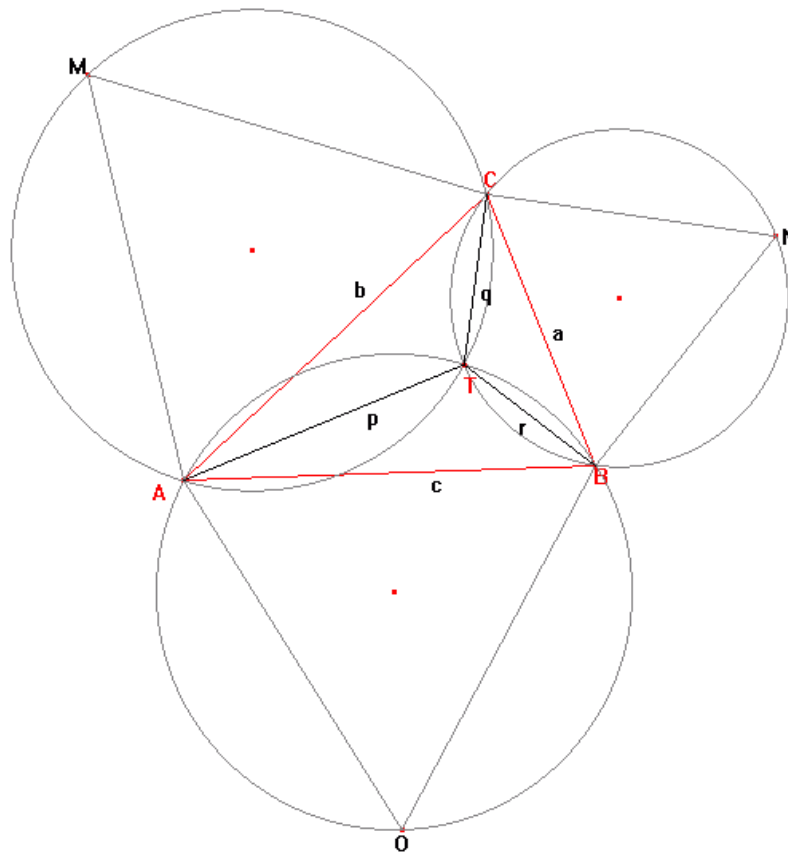
# Project Euler #143: Investigating the Torricelli point of a triangle

This problem is a programming version of [Problem 143](#) from [projecteuler.net](#)

Let  $ABC$  be a triangle with all interior angles being less than  $120$  degrees. Let  $X$  be any point inside the triangle and let  $XA = p$ ,  $XB = q$ , and  $XC = r$ .

Fermat challenged Torricelli to find the position of  $X$  such that  $p + q + r$  was minimised.

Torricelli was able to prove that if equilateral triangles  $AOB$ ,  $BNC$  and  $AMC$  are constructed on each side of triangle  $ABC$ , the circumscribed circles of  $AOB$ ,  $BNC$ , and  $AMC$  will intersect at a single point,  $T$ , inside the triangle. Moreover he proved that  $T$ , called the Torricelli/Fermat point, minimises  $p + q + r$ . Even more remarkable, it can be shown that when the sum is minimised,  $AN = BM = CO = p + q + r$  and that  $AN$ ,  $BM$  and  $CO$  also intersect at  $T$ .



If the sum is minimised and  $a$ ,  $b$ ,  $c$ ,  $p$ ,  $q$  and  $r$  are all positive integers we shall call triangle  $ABC$  a Torricelli triangle. For example,  $a = 399$ ,  $b = 455$ ,  $c = 511$  is an example of a Torricelli triangle, with  $p + q + r = 784$ .

Given  $N$ , print all the side lengths  $(a, b, c)$  of all Torricelli triangles having  $p + q + r \leq N$ . To ensure that no triangle is printed more than once, ensure that  $a \leq b \leq c$ . Print the triangles with smaller  $a$  first, and

in case of ties, smaller  $b$ s, and in case of ties, smaller  $c$ s.

### Input Format

The input contains a single integer,  $N$ .

### Constraints

Input file #1-#2:

$$1 \leq N \leq 10^4$$

Input file #3-#4:

$$1 \leq N \leq 10^5$$

Input file #5-#8:

$$1 \leq N \leq 4 \cdot 10^5$$

### Output Format

For each test case, output one line for each Torricelli triangle containing three integers separated by single spaces:  $a$ ,  $b$  and  $c$ .

### Sample Input

```
1000
```

### Sample Output

```
399 455 511
```

### Explanation

There is only one such triangle, which is described in the problem statement.