Project Euler #137: Fibonacci golden nuggets

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This problem is a programming version of Problem 137 from projecteuler.net

Consider the infinite polynomial series $A_F(x) = xF_1 + x^2F_2 + x^3F_3 + \ldots$, where F_k is the k^{th} term in the Fibonacci sequence: $1, 1, 2, 3, 5, 8, \ldots$; that is, $F_k = F_{k-1} + F_{k-2}$, $F_1 = 1$ and $F_2 = 1$.

For this problem we shall be interested in values of x for which $A_F(x)$ is a positive integer.

Surprisingly

$$\begin{aligned} A_F(1/2) &= (1/2)\cdot 1 + (1/2)^2\cdot 1 + (1/2)^3\cdot 2 + (1/2)^4\cdot 3 + (1/2)^5\cdot 5 + (1/2)^6\cdot 8 + \cdots \\ &= 1/2 + 1/4 + 2/8 + 3/16 + 5/32 + \ldots \\ &= 2 \end{aligned}$$

The corresponding values of x for the first five natural numbers are shown below.

x	$A_F(x)$
$\sqrt{2}-1$	1
$\frac{1}{2}$	2
$\frac{\sqrt{13}-2}{3}$	3
$\frac{\sqrt{89}-5}{8}$	4
$\frac{\sqrt{34}-3}{5}$	5

We shall call $A_F(x)$ a golden nugget if x is rational, because they become increasingly rarer; for example, the 10^{th} golden nugget is 74049690.

Given N, find the N^{th} golden nugget. Since this number can be very large, output it modulo $10^9 + 7$.

Input Format

The first line of input contains T, the number of test cases.

Each test case consists of a single line containing a single integer, N.

Constraints

$1 \le T \le 10^5$

In the first test case: $1 \le N \le 20$
In the second test case: $1 \le N \le 10^6$
In the third test case: $1 \le N \le 10^{18}$

Output Format

For each test case, output a single line containing a single integer, the answer for that test case.

Sample Input

```
2
1
10
```

Sample Output

```
2
74049690
```