Project Euler #65: Convergents of e

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This problem is a programming version of Problem 65 from projecteuler.net

The square root of 2 can be written as an infinite continued fraction.

$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cdots}}}}$$

The infinite continued fraction can be written, $\sqrt{2} = [1; (2)]$, (2) indicates that 2 repeats *ad infinitum*. In a similar way, $\sqrt{23} = [4; (1, 3, 1, 8)]$.

It turns out that the sequence of partial values of continued fractions for square roots provide the best rational approximations. Let us consider the convergents for $\sqrt{2}$.

$$1 + \frac{1}{2} = \frac{3}{2}$$

$$1 + \cfrac{1}{2 + \frac{1}{2}} = \frac{7}{5}$$

$$1 + \cfrac{1}{2 + \cfrac{1}{2 + \frac{1}{2}}} = \frac{17}{12}$$

$$1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \frac{1}{2}}}} = \frac{41}{29}$$

Hence the sequence of the first ten convergents for $\sqrt{2}$ are:

$$1, \frac{3}{2}, \frac{7}{5}, \frac{17}{12}, \frac{41}{29}, \frac{99}{70}, \frac{239}{169}, \frac{577}{408}, \frac{1393}{985}, \frac{3363}{2378}, \ldots$$

What is most surprising is that the important mathematical constant,

$$e = [2;\ 1, 2, 1,\ 1, 4, 1,\ 1, 6, 1, \cdots,\ 1, 2k, 1, \cdots]$$

The first ten terms in the sequence of convergents for e are:

$$2, 3, \frac{8}{3}, \frac{11}{4}, \frac{19}{7}, \frac{87}{32}, \frac{106}{39}, \frac{193}{71}, \frac{1264}{465}, \frac{1457}{536}, \ldots$$

The sum of digits in the numerator of the 10^{th} convergent is $1 + 4 + 5 + 7 = 17$.

Find the sum of digits in the numerator of the N^{th} convergent of the continued fraction for e.

Input Format

Input contains an integer N

Constraints

$1 \leq N \leq 30000$

Output Format

Print the answer corresponding to the test case.

Sample Input

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10
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Sample Output

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17
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