Project Euler #64: Odd period square roots

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This problem is a programming version of Problem 64 from projecteuler.net

All square roots are periodic when written as continued fractions and can be written in the form:

$$\sqrt{N} = a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \cdots}}}$$

For example, let us consider $\sqrt{23}$:

$$\sqrt{23} = 4 + \sqrt{23} - 4 = 4 + \frac{1}{\frac{1}{\sqrt{23}-4}} = 4 + \frac{1}{1 + \frac{\sqrt{23}-3}{7}}$$

If we continue we would get the following expansion:

$$\sqrt{23} = 4 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{8 + \cdots}}}}$$

The process can be summarised as follows:

$$\begin{aligned}
a_0 &= 4, \frac{1}{\sqrt{23}-4} &&= \frac{\sqrt{23}+4}{7} &&= 1 + \frac{\sqrt{23}-3}{7} \\
a_1 &= 1, \frac{7}{\sqrt{23}-3} &&= \frac{7(\sqrt{23}+3)}{14} &&= 3 + \frac{\sqrt{23}-3}{2} \\
a_2 &= 3, \frac{2}{\sqrt{23}-3} &&= \frac{2(\sqrt{23}+3)}{14} &&= 1 + \frac{\sqrt{23}-4}{7} \\
a_3 &= 1, \frac{7}{\sqrt{23}-4} &&= \frac{7(\sqrt{23}+4)}{7} &&= 8 + \sqrt{23} - 4 \\
a_4 &= 8, \frac{1}{\sqrt{23}-4} &&= \frac{\sqrt{23}+4}{7} &&= 1 + \frac{\sqrt{23}-3}{7} \\
a_5 &= 1, \frac{7}{\sqrt{23}-3} &&= \frac{7(\sqrt{23}+3)}{14} &&= 3 + \frac{\sqrt{23}-3}{2} \\
a_6 &= 3, \frac{2}{\sqrt{23}-3} &&= \frac{2(\sqrt{23}+3)}{14} &&= 1 + \frac{\sqrt{23}-4}{7} \\
a_7 &= 1, \frac{7}{\sqrt{23}-4} &&= \frac{7(\sqrt{23}+4)}{7} &&= 8 + \sqrt{23} - 4
\end{aligned}$$

It can be seen that the sequence is repeating. For conciseness, we use the notation $\sqrt{23} = [4; (1, 3, 1, 8)]$, to indicate that the block $(1, 3, 1, 8)$ repeats indefinitely.

The first ten continued fraction representations of (irrational) square roots are:

$\sqrt{2} = [1; (2)], period = 1$
$\sqrt{3} = [1; (1, 2)], period = 2$
$\sqrt{5} = [2; (4)], period = 1$
$\sqrt{6} = [2; (2, 4)], period = 2$
$\sqrt{7} = [2; (1, 1, 1, 4)], period = 4$
$\sqrt{8} = [2; (1, 4)], period = 2$
$\sqrt{10} = [3; (6)], period = 1$
$\sqrt{11} = [3; (3, 6)], period = 2$
$\sqrt{12} = [3; (2, 6)], period = 2$
$\sqrt{13} = [3; (1, 1, 1, 1, 6)], period = 5$

Exactly four continued fractions, for $x \le 13$, have an odd period.

How many continued fractions for $x \le N$ have an odd period?

Input Format

Input contains an integer N

Constraints

$10 \le N \le 30000$

Output Format

Print the answer corresponding to the test case.

Sample Input

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13
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Sample Output

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4
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